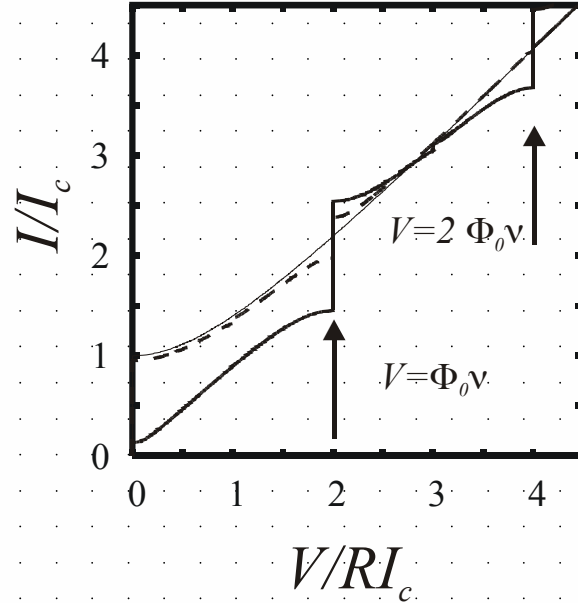
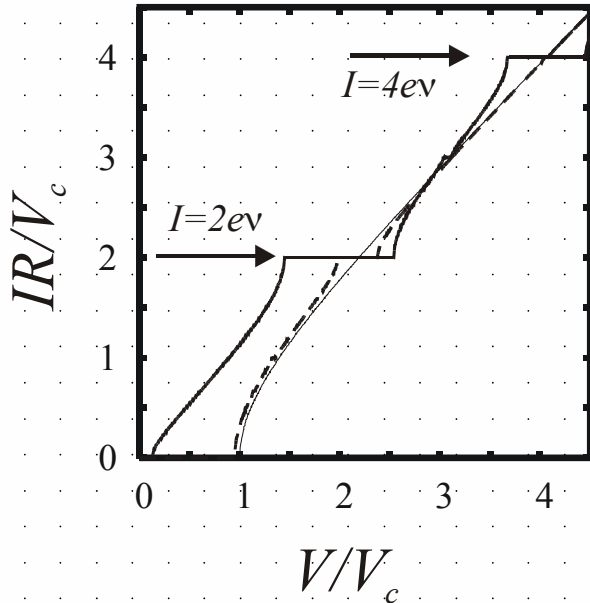
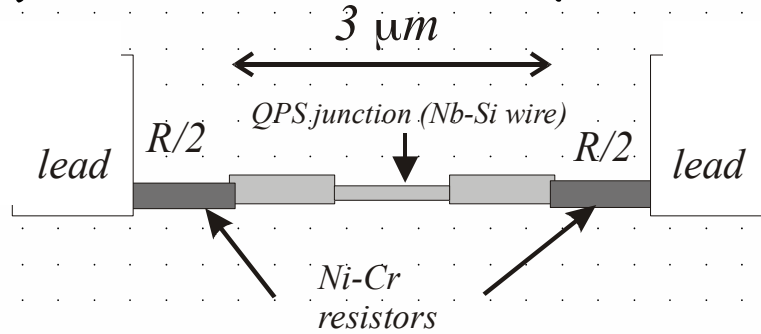


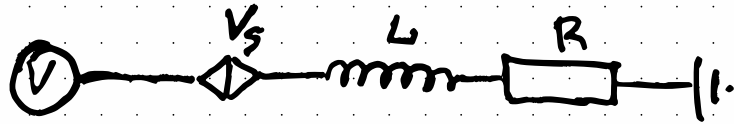
Quantum phase slips in double junction  
a current standard by gate modulation

Janis Erdmanis

# What is a quantum phase slip?



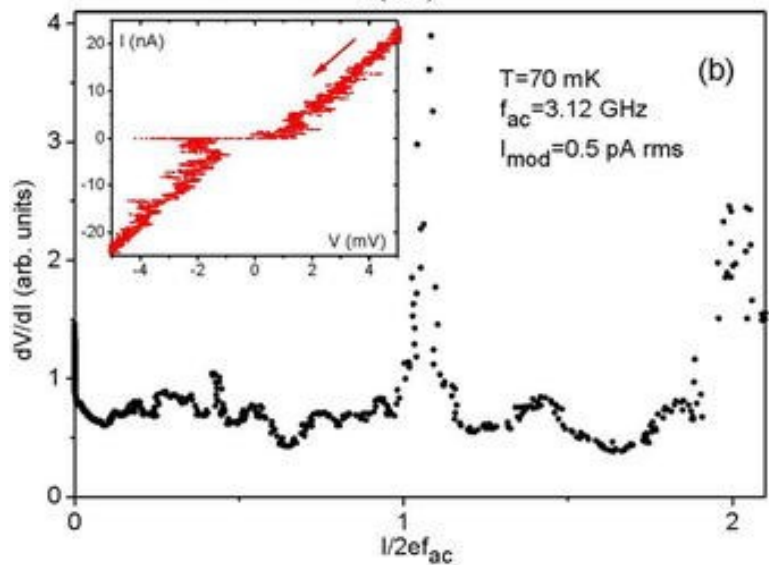
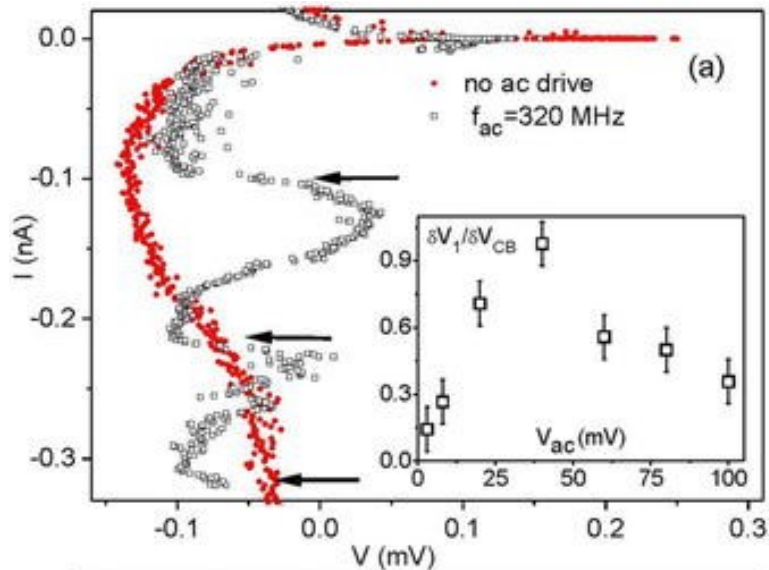
# Measuring a phase slip



$$V(t) = V_s \sin\left(\frac{\pi}{e} q\right) + L \frac{d^2 q}{dt^2} + R \frac{dq}{dt}$$

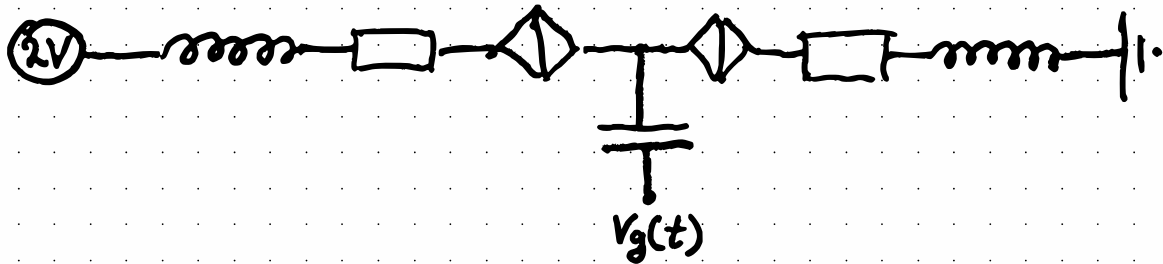
- $E_s \gg E_L$  for the charge to be a relevant quantum variable
- $R \gg L$  for non-hysteretic regime

$$\Rightarrow R = 60 \text{ k}\Omega \quad [1]$$



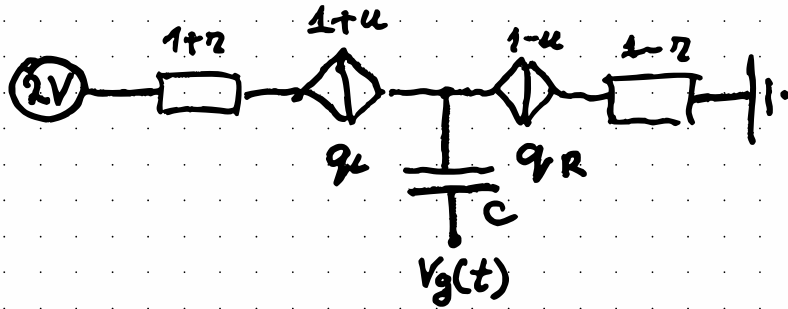
<https://aip.scitation.org/doi/10.1063/1.5092271>

# The double junction setup



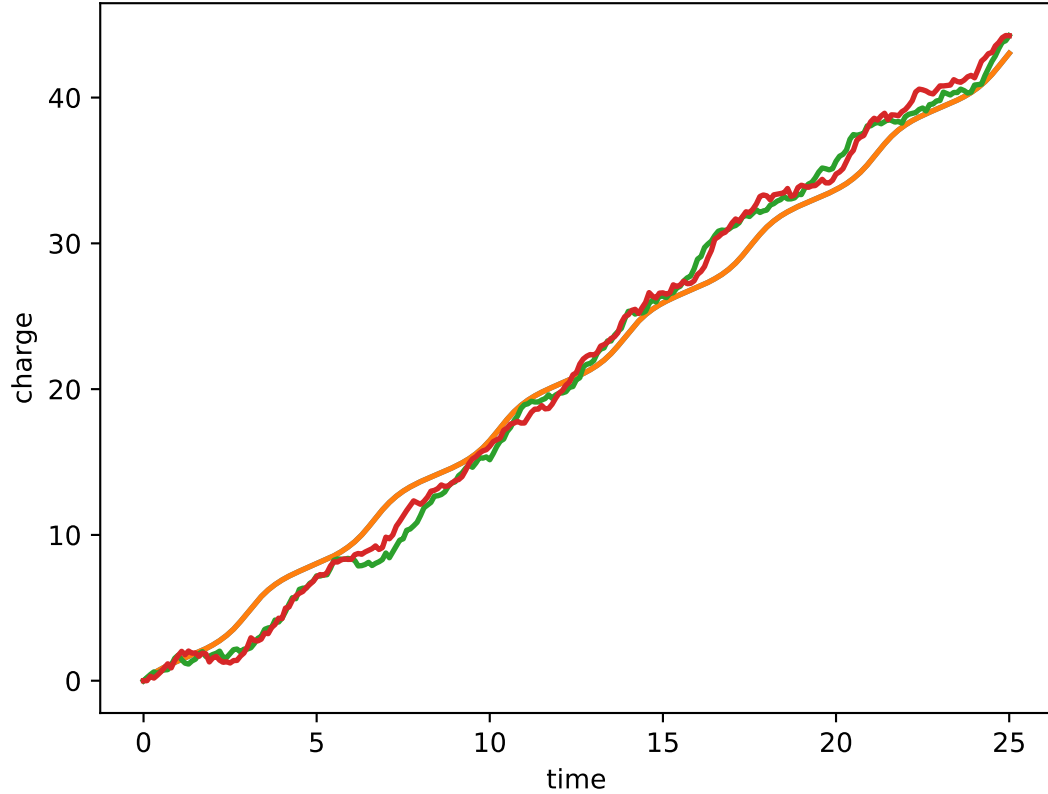
- Since  $V$  is constant we no longer need to fight inductivity with current
- The second phase slip allows us to alternate between constructive and destructive interference just by changing a gate voltage

The equations for double junction

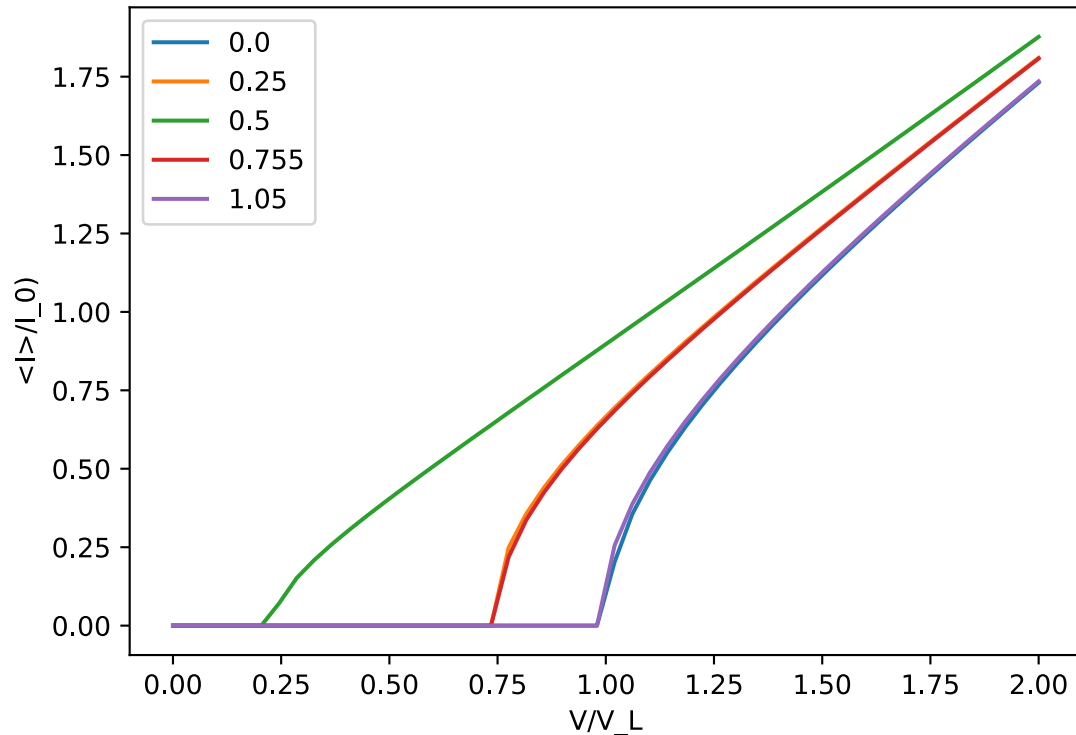


$$\begin{cases} \dot{q}_L = \frac{1}{1+r_2} (2V - (1+u) \sin q_L - \frac{1}{C} (q_L - q_R) - V_g(t)) + \text{Noise} \\ \dot{q}_R = \frac{1}{1-r_2} (-(1-u) \sin q_R + \frac{1}{C} (q_L - q_R) + V_g(t)) + \text{Noise} \end{cases}$$

# Numerical solution for charge

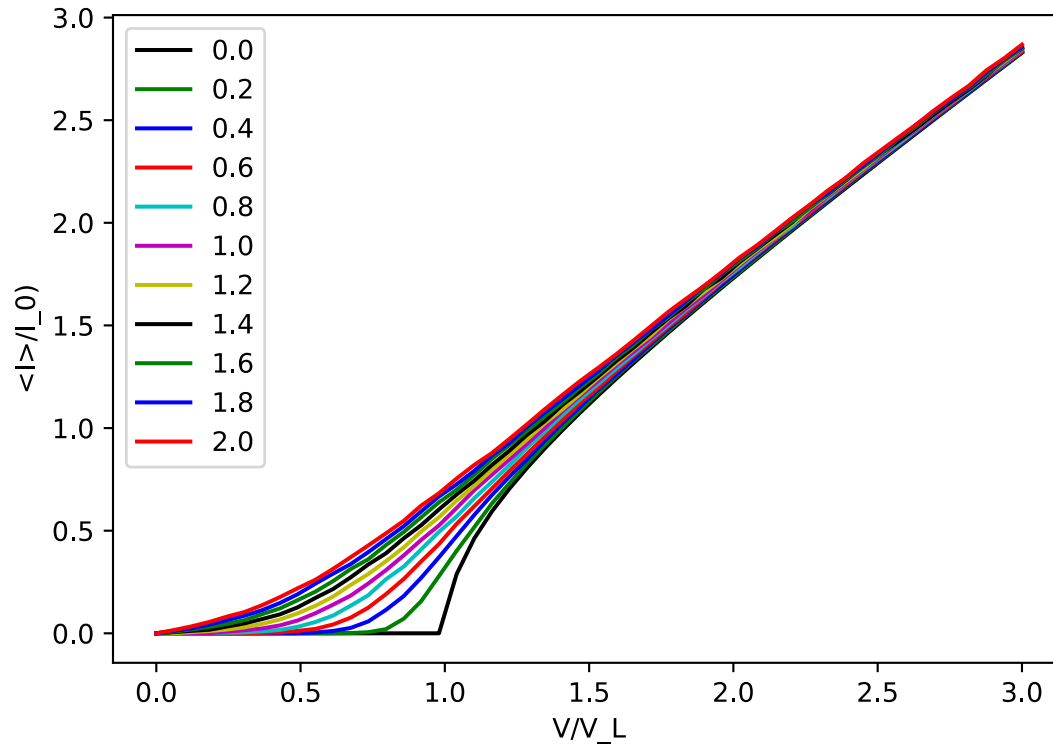


# Average current as function of gate voltage



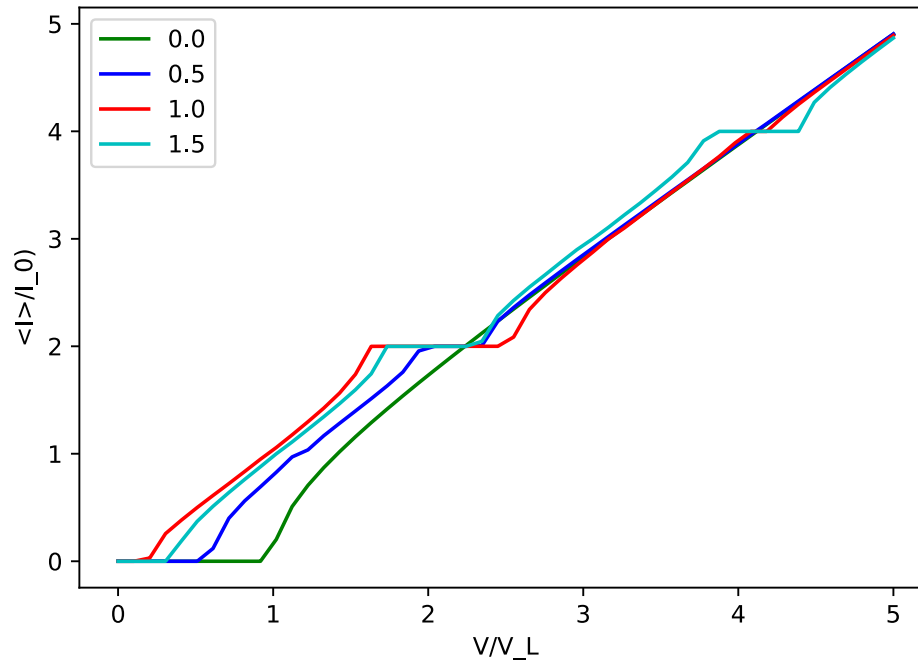


# Effect of temperature on double phase slip system

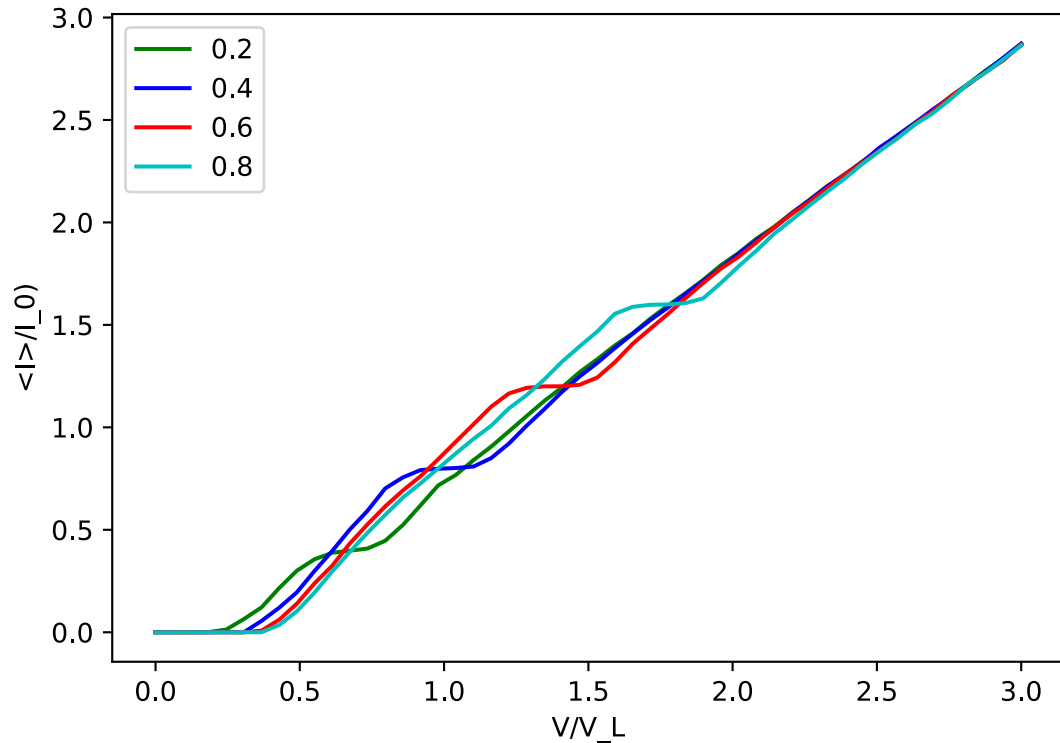


# Harmonic modulation of gate voltage

Any nonlinear system with a limit cycle modulated with a signal gives rise to Shapiro steps



The step at finite temperature and different frequencies



# The rise of Shaphire steps (theory)

$$\begin{cases} \dot{q}_L = \frac{1}{1+\tau} (2V - (1+u) \sin q_L - \frac{1}{C} (q_L - q_R) - V_g(t)) \\ \dot{q}_R = \frac{1}{1-\tau} (-(1-u) \sin q_R + \frac{1}{C} (q_L - q_R) + V_g(t)) \end{cases}$$

We shall change variables  $q_{LR} = q \pm \Delta q$  and consider a symmetric setup ( $u, \tau = 0$ )

$$\begin{cases} \dot{q} = V - \cos \Delta q \sin q \\ \Delta \dot{q} = \cos q \sin \Delta q - \frac{2}{C} (\Delta q - q_g(t)) \end{cases} \quad \begin{array}{l} \swarrow \\ \text{rescaled } V_g(t) \end{array}$$

If capacitance is small we can expect  $\Delta q = q_g(t)$ :

$$\dot{q} = V - \cos q_g(t) \sin q$$

Phase locking and equation for slowly varying phase

$$\dot{q}_* = V - \omega \cdot \sin q_*$$

The solution to this equation is time invariant and thus we have a degenerate solution

$$q_* = q_*(t + \frac{1}{\Omega} \varphi)$$

Now if we replace  $\omega \rightarrow \omega(t)$  we break the symmetry. That manifests in a slowly varying phase thus

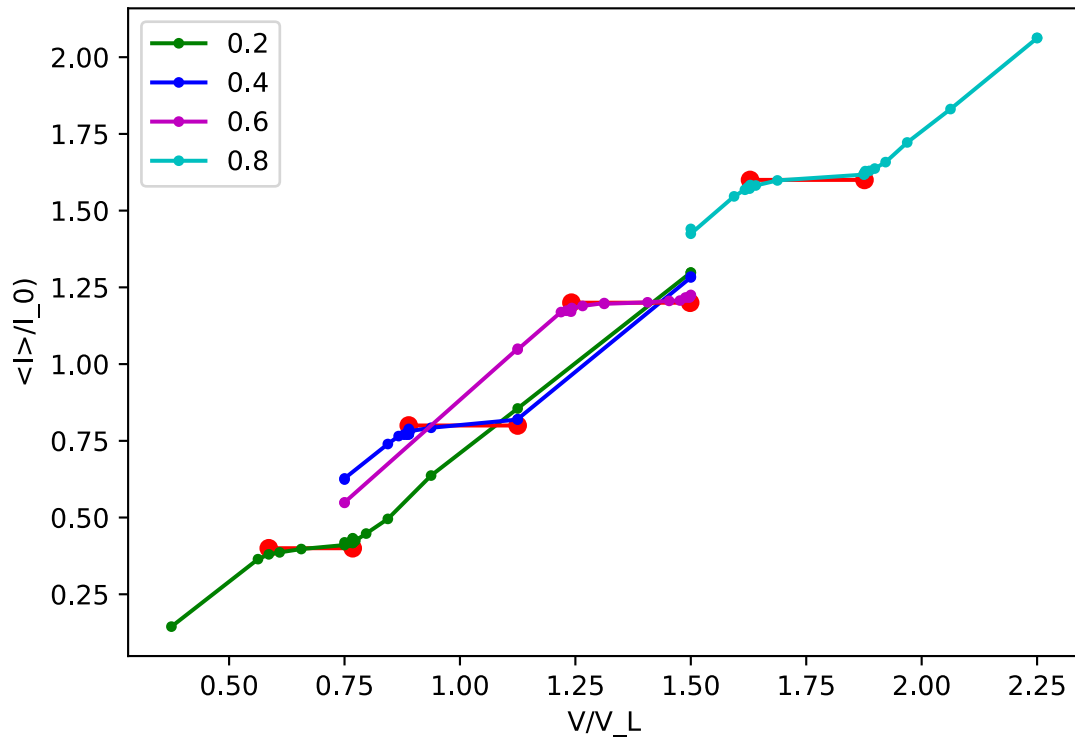
$$q(t) = q_*(t + \frac{1}{\Omega} \varphi(t))$$

Plugging it in equation for  $\dot{q}$  in case where  $\omega = \Omega$  we derive:

$$\dot{\varphi} = \delta V + \frac{\Delta V}{2} \sin \varphi$$

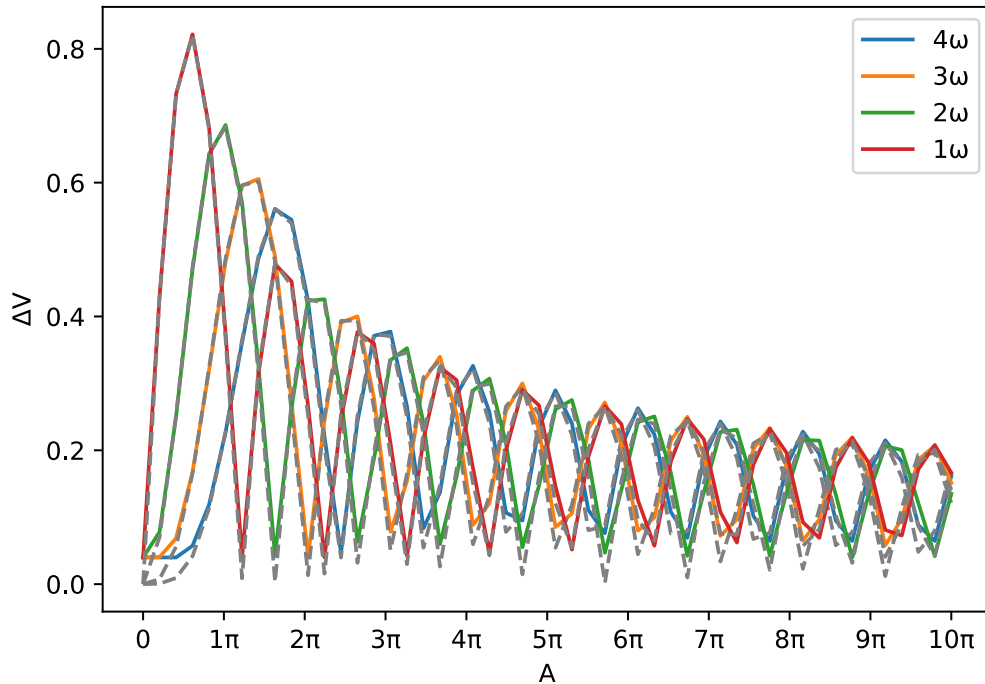
$$|\delta V| < \frac{\Delta V}{2} \Rightarrow \varphi = \varphi(\delta V) \quad \swarrow \text{a phase locking}$$

# The step extraction algorithm



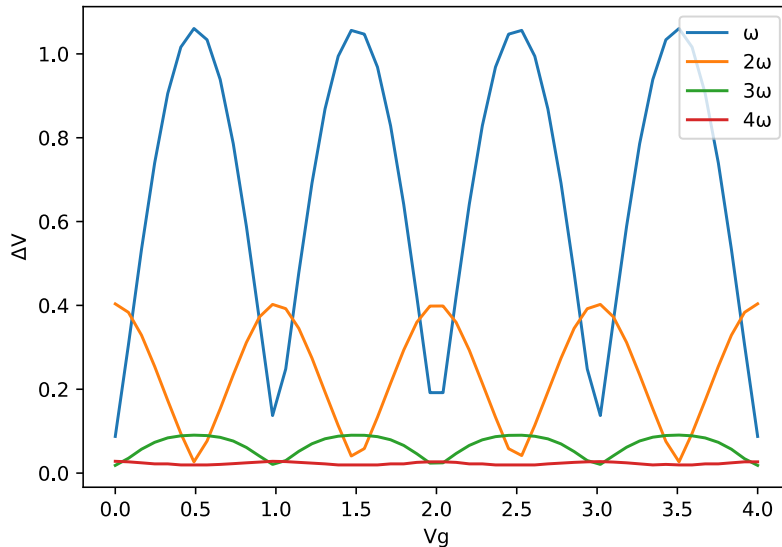
The step width as function of amplitude

$$\Delta V_{\kappa} = 2 S_{\kappa\omega, \Omega} \mathcal{H}_{\kappa}(A) \begin{cases} |\cos q_0| & \text{if } \kappa \text{ is even} \\ |\sin q_0| & \text{if } \kappa \text{ is odd} \end{cases}$$



The step width as function of gate voltage

$$\Delta V_{\kappa} = 2 S_{\kappa\omega, \Omega} \mathcal{F}_{\kappa}(\Gamma) \begin{cases} |\cos q_0| & \text{if } \kappa \text{ is even} \\ |\sin q_0| & \text{if } \kappa \text{ is odd} \end{cases}$$



For asymmetric setup this dependance factors away

$$|\cos q_0| \rightarrow \sqrt{\cos^2 q_0 + \mu \sin^2 q_0}$$

$$|\sin q_0| \rightarrow \sqrt{\sin^2 q_0 + \mu \cos^2 q_0}$$



# The raise of fractional steps

$$\begin{cases} \dot{q} = v - \cos \Delta q \sin q \\ \Delta \dot{q} = \cos q \sin \Delta q - \frac{2}{c} (\Delta q - q_g(t)) \end{cases}$$

Previously we assumed that  $\Delta q = q_g(t)$ . If capacitance is finite we shall consider correction  $\Delta q = q_g(t) + q_c(t)$ .

$$\begin{aligned} \Delta q: \quad \dot{q}_g + \dot{q}_c &= -\cos q \sin(q_g + q_c) - \frac{2}{c} q_c \\ q_c &= -\frac{c}{2} (\dot{q}_g + \sin q_g \cdot \cos q) + O(c^2) \end{aligned}$$

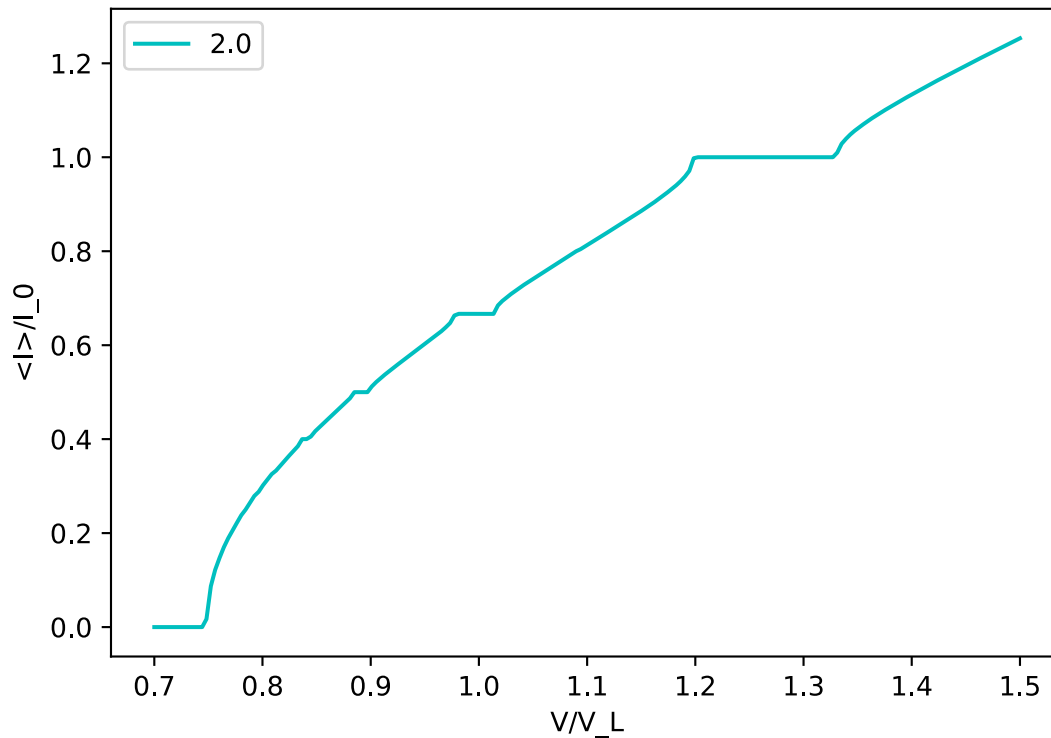
Putting that in equation for  $q$  gives us:

$$\dot{q} = v - \cos q_g(t) \cdot \sin q - \frac{c}{4} \sin^2 q_g(t) \cdot \sin 2q$$

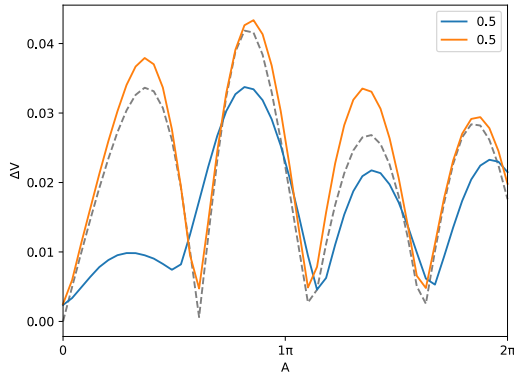
which makes equation for  $q(t)$  (if  $N\Omega = M\omega$ ):

$$\dot{\varphi} = \delta V + \sum_n b_n e^{in\varphi}; \quad \Delta V = (\max_{\varphi} - \min_{\varphi}) \left( \sum_n b_n e^{in\varphi} \right)$$

# The fractional steps in numerics



# Ongoing research



- Finish deriving a formula for fractional steps
- Look into large capacitance limit.

